

# Software for Evaluating Relevance of Steps in Algebraic Transformations

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Synt. err. 0 Ord. err. 0  
Transf. err. 0 Answ. err. 0

Task: Find the full disjunctive normal form of the formula  
Action: Choose a formula for transformation and apply a suitable rule

$(A \& \neg C \sim B) \vee C \supset C \equiv$   
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C \equiv$   
 $(A \& \neg C \& B \vee (\neg A \vee \neg C) \& \neg B) \vee C \supset C \equiv$   
 $(A \& \neg C \& B \vee (\neg A \vee C) \& \neg B) \vee C \supset C \equiv$   
 $\neg((A \& \neg C \& B \vee (\neg A \vee C) \& \neg B) \vee C) \vee C \equiv$   
 $\neg((A \& \neg C \& B \vee \neg A \& \neg B \vee C \& \neg B) \vee C) \vee C \equiv$   
 $\neg((A \& \neg C \& B \vee \neg A \& \neg B \& C \vee \neg A \& \neg B \& \neg C \vee C \& \neg B) \vee C) \equiv$   
 $\neg(A \& \neg C \& B \vee \neg A \& \neg B \& C \vee \neg A \& \neg B \& \neg C \vee C \& \neg B) \& \neg C \vee C$

$(X)$	$\Rightarrow$	$\Leftarrow$	$X$	$X \supset Y$	$\Rightarrow$	$\Leftarrow$	$\neg(X \& \neg Y)$	$X \& (Y \vee Z)$	$\Rightarrow$	$\Leftarrow$	$X \& Y \vee X \& Z$
$\neg X$	$\Rightarrow$	$\Leftarrow$	$X$	$\neg(X \supset Y)$	$\Rightarrow$	$\Leftarrow$	$X \& \neg Y$	$X \vee Y \& Z$	$\Rightarrow$	$\Leftarrow$	$(X \vee Y) \& (X \vee Z)$
$X \& Y$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$X \supset Y$	$\Rightarrow$	$\Leftarrow$	$\neg X \vee Y$	$\neg X \& X \vee Y$	$\Rightarrow$	$\neg Y$	
$\neg(X \& Y)$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$X \sim Y$	$\Rightarrow$	$\Leftarrow$	$X \& Y \vee \neg X \& \neg Y$	$(\neg X \vee X) \& Y$	$\Rightarrow$	$\neg Y$	
$X \vee Y$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$\neg(X \sim Y)$	$\Rightarrow$	$\Leftarrow$	$X \& \neg Y \vee \neg X \& Y$	$X \vee X \& Y$	$\Rightarrow$	$X$	
$\neg(X \vee Y)$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$X \sim Y$	$\Rightarrow$	$\Leftarrow$	$(X \supset Y) \& (Y \supset X)$	$X \& (X \vee Y)$	$\Rightarrow$	$X$	
$X \& Y$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$\neg(X \sim Y)$	$\Rightarrow$	$\Leftarrow$	$\neg(X \supset Y) \vee \neg(Y \supset X)$	$X \oplus Y$	$\Rightarrow$	$Y \oplus X$	
$\neg(X \& Y)$	$\Rightarrow$	$\Leftarrow$	$X$	$X$	$\Rightarrow$	$\Leftarrow$	$X \& Y \vee X \& \neg Y$	$X \oplus X$	$\Rightarrow$	$X$	
$X \vee Y$	$\Rightarrow$	$\Leftarrow$	$\neg Y$	$X$	$\Rightarrow$	$\Leftarrow$	$(X \vee Y) \& (X \vee \neg Y)$	$X \oplus (Y \oplus Z)$	$\Rightarrow$	$(X \oplus Y) \oplus Z$	

- 1: Stage 1:  
Rule applied: 15: biconditional eliminated - OK  
 $(A \& \neg C \sim B) \vee C \supset C$   
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$
- 2: Stage 1:  
Rule applied: 4: Negation into & or v  
First eliminate bicond-s and impl-s!  
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$   
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$
- 3: Stage 1:  
Rule applied: 2: Cancel double negation - OK  
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$   
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$
- 4: Stage 1:  
Rule applied: 13: impl eliminated - OK  
 $(A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C \supset C$   
 $\neg((A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C) \supset C$
- 5: Stage 2:  
Rule applied: 21: multiplication of disj-s  
Stage 2 not yet finished (negations into brackets)!  
 $\neg((A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C) \supset C$   
 $\neg((A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C) \supset C$
- 6: Stage 2:  
Rule applied: 19: adding variables by rule  $X \rightarrow X \& Y \vee X \& \neg Y$   
This conversion belongs to stage 5!  
 $\neg((A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C) \supset C$   
 $\neg((A \& \neg C \& B \vee \neg(A \& \neg C) \& \neg B) \vee C) \supset C$