Implicit Content Dictionaries in the NIST Digital Repository of Mathematical Formulae

Moritz Schubotz
with material from Howard Cohl

NIST Digital Repository of Mathematical Formulae Goals

- DRMF compendium of individual formulae for a mathematically literate audience
  1. OPSF **community interaction** for mathematicians and scientists
  2. **expandability** from the literature (support the DLMF)
  3. context-free full **semantic** information
  4. **user friendly** perspective
  5. **searchable** mathematics
  6. modern **MathML** tools

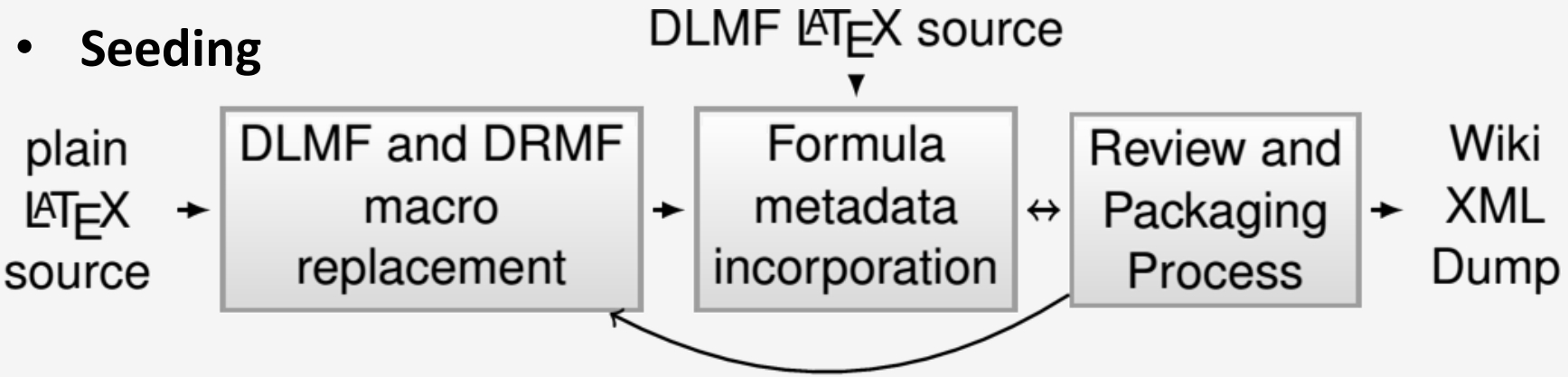
First three DRMF seeding projects

- **2013** 167 DLMF Formulae Home Pages
  - 533 semantic DLMF Macros (developed by Bruce Miller)
  - Content MathML with LaTeXML‘s implicit Content Dictionaries
- **2014** 1469 additional KLS (OP) Formulae Home Pages
  - Additional 153 semantic DRMF macros
- **2015** ~5000 eCF, BMP Formula Home Pages
  - Non LaTeX input

Current DRMF Implementation

- **Platform**: MediaWiki with Math and MathSearch extensions
  - 2 Table of Contents Pages
  - 76 Lists of Formulas Pages
  - 124 Definition Pages for DRMF macros
  - 1636 Formula Home Pages

- **Seeding**



(Bruce Miller) LaTeXML (DLMF) Semantic LaTeX macros

Name	Type	Rendering	LaTeX	semantic LaTeX
Trigonometric sine	f	$\sin z$	`\sin z`	`\sin@@{z}`
Euler gamma	f	$\Gamma(z)$	`\Gamma(z)`	`\EulerGamma@{z}`
Jacobi polynomial	p	$P_n^{(\alpha,\beta)}(x)$	`P_n^{(\alpha, \beta)}(x)`	`\Jacobi{\alpha}{\beta}{n}@{x}`
little q-Laguerre polynomial	p	$p_n(x;a\|q)$	`p_n(x;a|q)`	`\littleqLaguerre{n}@{x}{a}{q}`

Semantic macro breakdown

- **689 semantic LaTeX macros (unpublished)**
  - 533 macros from DLMF
  - 156 additional DRMF macros using the LaTeXML framework
- **Semantic LaTeX macro properties**:
  - lengths between 1 and 26 characters (median length is 8 characters)
  - Strict Naming conventions
    - individual's names capitalized
    - abbreviations utilized
    - macro names correspond with object names
    - Roman numerals

Macro Replacements for Generic LaTeX Source Datasets

- For the 3 chapters of KLS (OP) as well as the KLSadd LaTeX source
- **89** semantic macros were replaced
- a total of **3308** times
- represented by **774** (LOC) of regular expressions written by Cherry Zou
- Currently the six most common replacements are:

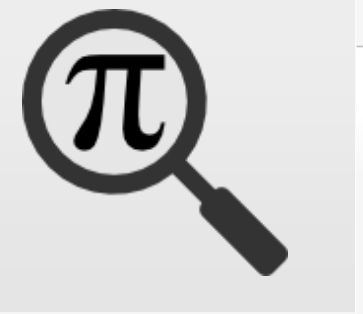
1. *q*-Pochhammer symbol – replaced **659** times
2. Euler gamma function – replaced **266** times
3. *q*-hypergeometric function – replaced **237** times
4. Pochhammer symbol – replaced **205** times
5. Racah polynomial – replaced **117** times
6. cosine function – replaced **82** times

Example of complexity of the problem – KLSadd dataset

- After processing of the LaTeX input, only formulas remain.
- Current metadata: are constraint and substitution annotations.
- Future metadata
  - bibliographic metadata
  - references to KLS formulae
  - errata information
  - formula comments and notes
  - symmetries in parameters
  - proofs, etc...

Semantic LaTeX Macro Glossary – csv format

- For each macro we store:
- Example Macro calling sequence
- Name of object described by macro
- Object description
- Brief summary and description of calling options
- Link to URL giving precise definition
- Glossary.csv used in generation of symbols lists within Wikitext and for statistical purposes.



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Formula:DLMF:25.2:E9

$$\zeta(s) = \sum_{k=1}^{N} \frac{1}{k^s} + \frac{N^{1-s}}{s-1} - \frac{1}{2}N^{-s} + \sum_{k=1}^{n} \binom{s+2k-2}{2k-1} \frac{B_{2k}}{2k} N^{1-s-2k} - \binom{s+2n}{2n+1} \int_N^{\infty} \frac{\widetilde{B}_{2n+1}(x)}{x^{s+2n+1}} \, \mathrm{d}x$$

Constraint(s)

$\Re\, s > -2n \;\&\; n, N = 1, 2, 3, \ldots$

Proof

We ask users to provide proof(s), reference(s) to proof(s), or further clarification on the proof(s) in this space.

Follows from

$$\zeta(s) = \sum_{k=1}^{N} \frac{1}{k^s} + \frac{N^{1-s}}{s-1} - s \int_N^{\infty} \frac{x - \lfloor x \rfloor}{x^{s+1}} \, \mathrm{d}x$$

by repeated integration by parts.

Symbols List

& : logical and

ζ : Riemann zeta function : http://dlmf.nist.gov/25.2#E1

Σ : sum : http://drmf.wmflabs.org/wiki/Definition:sum

$\binom{n}{k}$: binomial coefficient : http://dlmf.nist.gov/1.2#E1 http://dlmf.nist.gov/26.3#SS1.p1

B_n : Bernoulli polynomial : http://dlmf.nist.gov/24.2#i

$\int$: integral : http://dlmf.nist.gov/1.4#iv

$\widetilde{B}_n$: periodic Bernoulli functions : http://dlmf.nist.gov/24.2#iii

$\mathrm{d}^n x$: differential : http://dlmf.nist.gov/1.4#iv

$\Re\, z$: real part : http://dlmf.nist.gov/1.9#E2

Bibliography

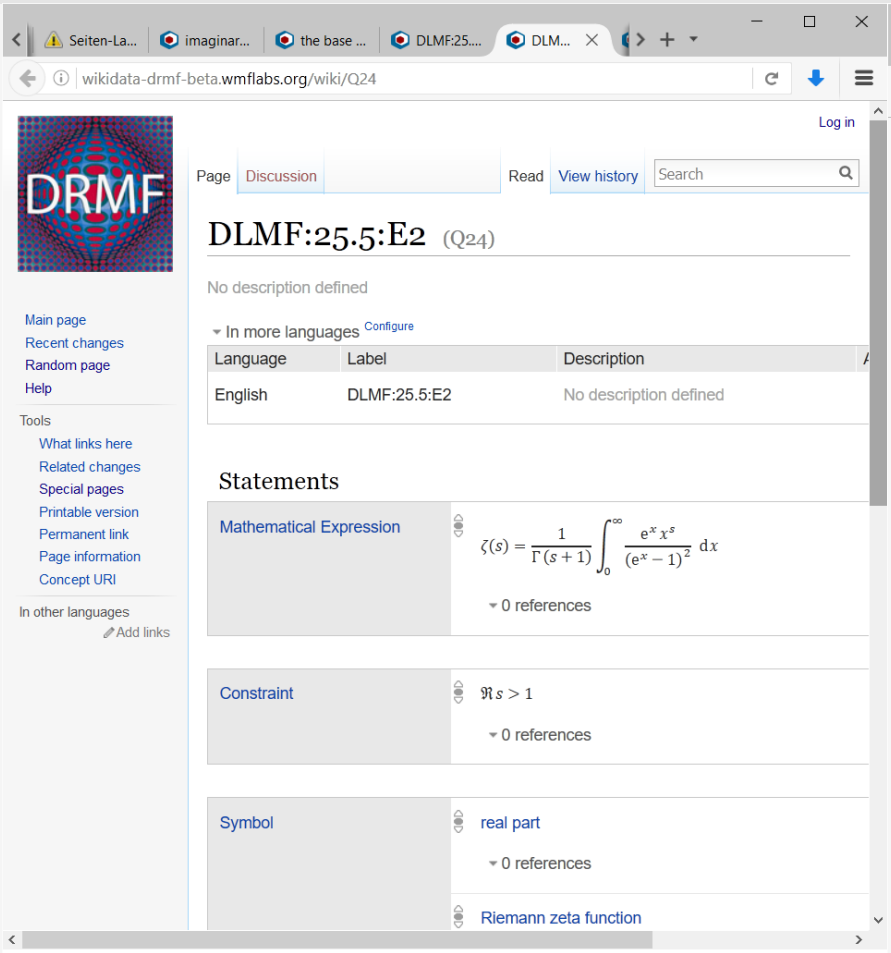
Equation (9), Section 25.2 of **DLMF**.

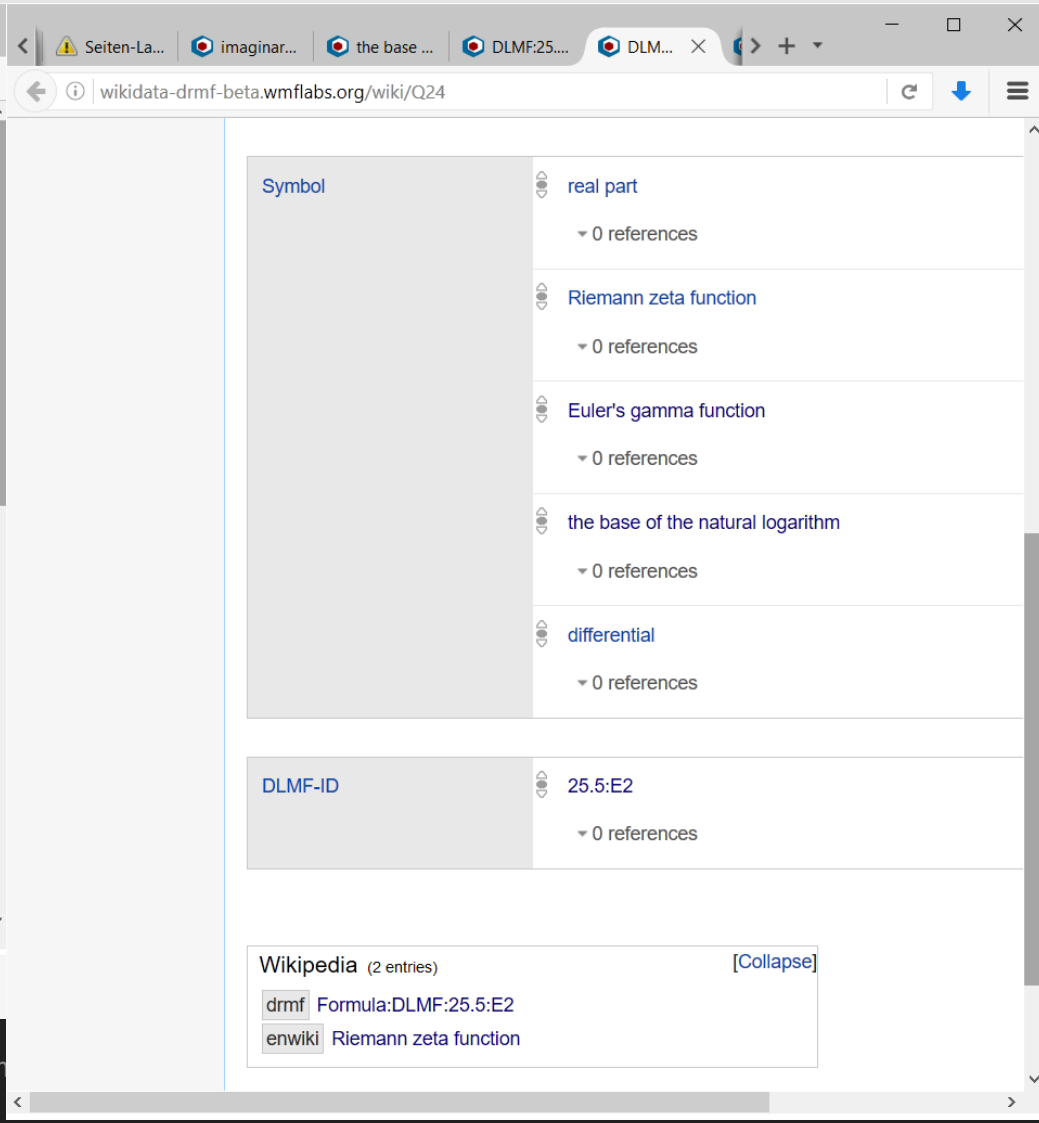
URL links

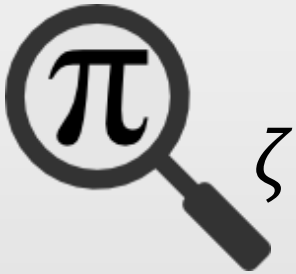
We ask users to provide relevant URL links in this space.

<< Formula:DLMF:25.2:E8 | formula in Definition and Expansions | Formula:DLMF:25.2:E10 >>

Transition to Wikibase storage







$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{\Gamma(n+s)}{n!\,\Gamma(s)} \zeta(n+s)\,(1-a)^n$$

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Contact

Moritz Schubotz (now at Universität Konstanz)

moritz@schubotz.de

+49 7531 88 4438

Mobile: +49 1578 047 1397

www.isg.uni-konstanz.de

www.formulasearchengine.com