CICM, Calculemus, MKM, GMDL, ...

Some Thoughts About Past and Future

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Sorry

My problem started in ... ~ 1963 ...

~ 1963

Thesis topic was in foundational research (poly ideals and algorithms) ⟶ Gröbner bases (1965, PhD)

I had to earn my living as a programmer (ZUSE Z 23) ⟶ applications in science and engineering

My problem (chance, challenge, opportunity, luck, ...)

All my life I lived, in parallel,

in the blue esoteric world of foundational mathematics

and in the red brutal world of applications

~ 1966 - 1974

the blue world: abstract problems, theorems, algorithms, proofs, ... formal aspects, logic

the red world: zoology, chemistry, medicine, ... soccer clubs, ski lifts, ...

1974 - : JKU

the blue world: ... , 1985 J Symbolic Computation, 1987 RISC, ...

the red world: local companies, ..., Softwarepark 1989, RISC Software, Univ of Applied Sciences, ... industrial projects, business, founding companies, creating jobs, attracting companies, ...

Combing blue and light blue

~ 1961: I was not satisfied with the quality of proofs, started to study the “yellow books”.

~ 1976: started to teach mathematics for computer science students with a radically new approach: first semester = predicate logic as a working language (Springer, 1979).

Started to “dream about CICM etc.”: create a system for doing mathematics and meta-mathematics in the same system (in “natural style”)

A controversy on "Doing Mathematics by Computer"

S. **Wolfram**'s Mathematica book, **1988**: "Mathematica - Doing Mathematics by Computer."

BB paper at DISCO **1993**: "Mathematica - Doing Mathematics by Computer **?** "

Controversy on what "doing mathematics" means:

> Wolfram: "I am amazed that such an obsolete notion like ***proof*** is still prevailing in mathematics."
>
> BB: "Mathematics is the arts of gaining knowledge by reasoning (***proving***, simplifying, solving)."

1995: Started to design **Theorema** with Mathematica as implementation language. Why Mathematica? First sketch at FROCOS 1996.

1996: A couple of groups like ours started the **Calculemus** initiative, conferences, projects.

1998: Stephen invited me for a keynote "Theorem Proving for the Masses" at his "10 Years of Mathematica" Conference.

(**Reconciliation**: Mathematica contains more and more theorem provers!)

~ 1996 -: The Calculemus dream

("-emus" Lat., a wish, a dream ..., what was the dream?)

Automate, computer-support, ... mathematical reasoning.

Early heroes: J. Herbrand (1930), resolution (A. Robinson, 1965), Mizar (A. Trybulec) 1973.

Today, 2018, at CADE / IJCAR: Dozens of very different automated reasoners with very different philosophies and techniques.

~ 1996 -: The Calculemus dream

The **Calculemus Dream ~ 1996 (**in my personal perspective):

- computer-supported mathematical *theory exploration* # automated proof of isolated theorems
- automated *proofs generation* # automated proof checking
- *theory dependent* reasoning methods # universal methods
- generation of "*readable*" proofs in "natural style"
- reasoning as interaction of *proving, solving, simplifying*
- *object and meta* layer in one system
- ....

Status: Much has been achieved. However, shame on me, when I "do" mathematics, I do it formally but not computer-supported.

~ 2001 - : The MKM dream

First MKM conference 2001 at RISC. (BB, G. Gonnet, M. Hazewinkel. Spec. issue of Annals of Math and AI, 2003.)

(MK)M, not M(KM) !

BB, Preface of MKM 2001 Proceedings (Annals of Math and AI, 2003):

Much time, effort, and money is being spent on inventing and proving new mathematical results. This effort is well spent for a discipline that forms the core of our current technology-based society. In fact, impressive progress has been made over the past decades in expanding the depth and breadth of our mathematical knowledge.

However, a decent part of our time, effort, and money is spent inefficiently because the knowledge added is hard to retrieve and, in fact, very little time, effort, and money is currently spent for improving the management of mathematical knowledge.

Of course, the advancements of computer technology, in particular the global web, promise to give us new tools of unprecedented power for making knowledge retrieval much more efficient and, of course, we already feel the practical and pleasant implications of this in our daily work when, by doing a few clicks, we get access to a huge amount of relevant information whose retrieval would have taken us many hours in libraries etc. just a couple of years ago.

However, I think that the urgently necessary improvement of mathematical knowledge management is essentially not a technological question but rather a deeply mathematical question and, in fact, a question of improving our abilities to do formal mathematics. I believe that mathematical knowledge management will turn out to be one of the most exciting future topics of mathematics and will lead to a new understanding of the fundamentals of mathematics in the same way as the foundational problems of mathematics in the early 20th century lead to a new, and much deeper, understanding of mathematics and to a whole wave of new directions, techniques and results. The impacts of advances in mathematical knowledge management on all of science and technology will be dramatic both because of the role of mathematics as the universal "thinking technology" of all science and technology and because techniques to be worked out for mathematical knowledge management will be applicable also for other, less structured, disciplines.

I also think that significant improvement in mathematical knowledge management will only be possible if the next generation of mathematicians reaches a much higher level in mastering the formal aspects of our field. Thus mathematical knowledge management is neither only a question of computer technology nor only a mathematical question but also a question of the sociology of the mathematical community. I am deeply convinced that ample time must be reserved for a thorough practical training of students in the formal and methodological aspects of mathematics in future math and computer science curricula. Only by combining the best tools of computer technology with a new and deep understanding and mastering of the structural formal fabrics of mathematics and the improved formal training of mathematicians will it be possible to reach a new level of accessibility and, hence, usefulness of mathematics.

Because of the high topicality of the emerging field of mathematical knowledge management, I

decided to guest-edit a special issue on this topic for the Annals of Mathematics and Artificial Intelligence and, in preparation of this special issue but organizationally independently, to organize an international workshop at RISC. I think that, as far as I know, this workshop is the first international event explicitly devoted to the topic of mathematical knowledge management in general and I am very happy that so many researchers representing so many different approaches are following our invitation. I am looking forward to presentations and discussions that will produce new interactions, ideas, projects and joint political activities for fast advancements in the next few years. In fact, some participants already proposed to host the future second and further editions of the MKM workshop. I think this is great and shows how timely and topical the workshop is.

Let me also take the opportunity to thank those who contributed to making the workshop possible, most prominently to Olga Caprotti, Christian Vogt, and Betina Curtis who basically structured, supervised and did all the preparatory work for this workshop so that I just could enjoy the excitement of the mathematical aspect of the field.

Bruno Buchberger

~ 2001 - : The MKM dream

Beyond Calculemus:

- formal reasoning only one tool; aspiration goes further
- logical and organizational management of entire mathematical theories
- translation between various formal presentations of math; translation of informal to formal math
- combining math theories from various systems
- accompanying the entire math exploration cycle (modeling, knowledge retrieval, invention, verification, archiving, ...)
- math specific knowledge management tools
- ....

Status: Much has been achieved but, shame on me, when I want to find math knowledge, I use “Google” (-like tools).

~ 2014 - : The GMDL (Global Math Digital Library) dream

2014 World Congress of Math (ICMS): Working Group GMDL was installed (the **"Friday Afternoon Club"**).

Great aspiration: Using all the technologies elaborated in Calculemus, MKM, ... (reasoning tools like Isabelle, ..., libraries like EuDML, ..., algorithm libraries like Maple, ..., standardization tools like MathML, ...):

- bring the global math knowledge into a coherent, *one-stop accessible* form
- *coarse grain*: "a library of papers", mainly based on natural language texts and phrases, like in other sciences
- *fine grain:* "a library of fomulae", based on formal presentation of math (coherent formal theories)
- *numerous tools* for coarse and fine grain contents
- with a "*world agency for math knowledge*" behind it taking care of the build-up and permanent expansion of the "GMDL"

~ 2014 - : The GMDL (Global Math Digital Library) dream

Status: Big efforts but I doubt we really moved significantly beyond the situation in 2014. ?? Discussion ...

My personal opinion: GMDL will only work with a **professionally organized and long-term financed organization** (kind of GMDL software company).

Stephen W got quite impatient about this and, within Mathematica, came up with something tangible:

```
MathematicalFunctionData["Sin",
 "IntegralRepresentations"]
```

$$\left\{\text{Function}\left[\{\dot{z}\}, \text{Inactivate}\left[\text{Sin}[\dot{z}] == \dot{z}\int_0^1 \text{Cos}\left[\dot{z}\,\dot{t}\right] d\dot{t}\right]\right]\right\}$$

```
FormulaData["KineticEnergy"]
```

$$K == \frac{1}{2} m v^2$$

My own (unsuccessful) efforts

2014 - 2016: I focused on **fund raising** (which also needs a lot of blue thinking) for installing such a GMDL Software Company.

- "Big grant application" EuKIM (12 groups) at EU HORIZON INRAIA programm: very good reports but "not good enough".
- Austrian Science Foundation: "We do not pay for 'infrastructure'. "
- Austrian Ministry for Research: "One of the best proposals we ever saw in the area but we have nothing to say ..."
- Austrian Academy of Science (I helped building up RICAM!): "Not really our focus .."
- JKU LIT Program: Four excellent reports "but not important enough for JKU."

Thus, at the end of 2016, **I decided to give up**. (And go back to the blue part of my life - and some extremely red aspects.)

My view on the future of "automated" math

I still consider the endeavor **extremely worthwhile**, namely

A. more and more sophisticated **automated reasoning methods integrated in systems** for all phases of "doing" mathematics, and

B. a **global "portal" for accessing mathematical knowledge** (both coarse grain and fine grain) sustained by a professional global publically funded company.

C. **much better formal training** in math and computer science curricula.

"Proof" of A: In 2004, I managed to "**simulate myself**". (By implementing two important principles of mathematical invention: **learning from failures** and thinking in **schemata**.)

"Proof" of B: DM Wang (**China**), my former postdoc, is thinking of doing something of that kind. (Shame on EU and US...)

"Proof" of C: spend the effort for improving the formal quality of math knowledge $\Longrightarrow$ save effort for retrieving math knowledge

A global math portal (sketch)

See attached file “BB: A global math portal”

What do you think?

B. Buchberger, 2016, EMS Berlin: Sketch of Global Math Portal