Towards an implementation in LambdaProlog of the two level Minimalist Foundation

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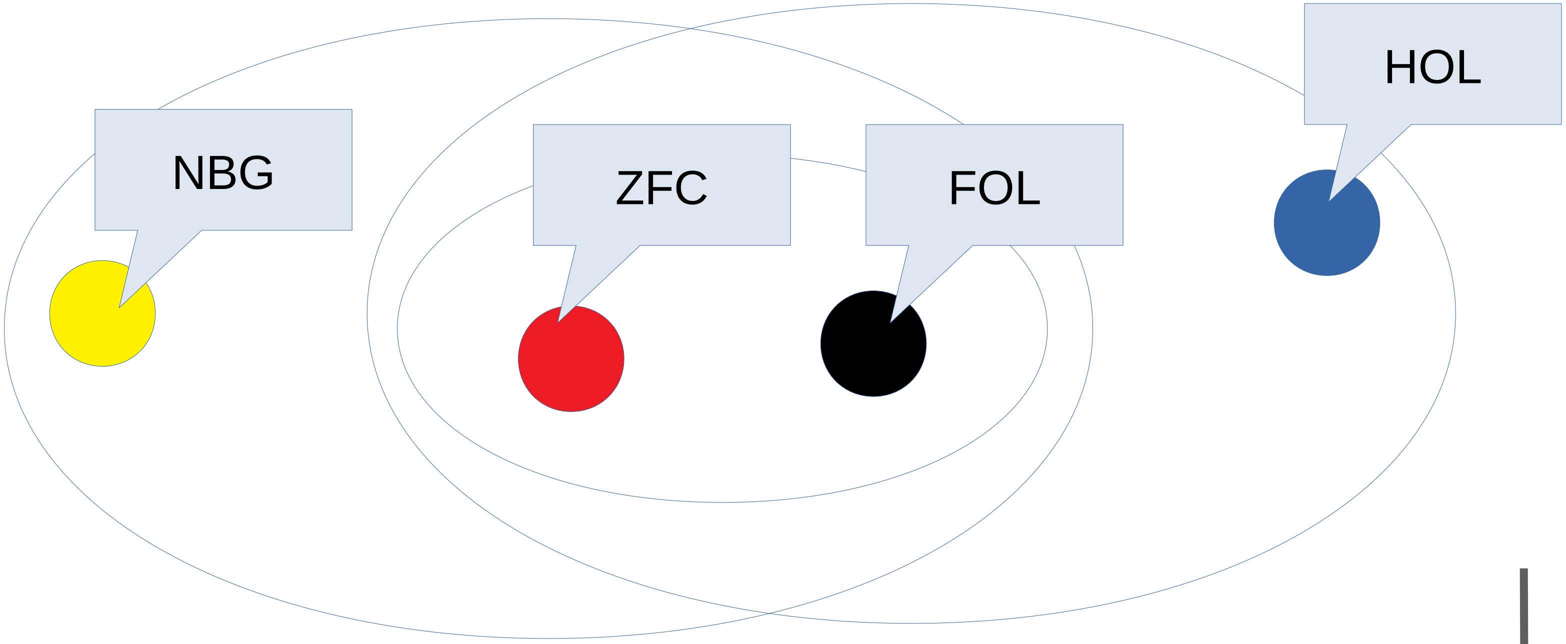
The
Minimalist Type Theory (MTT)
of Maietti and Sambin

The Classical World

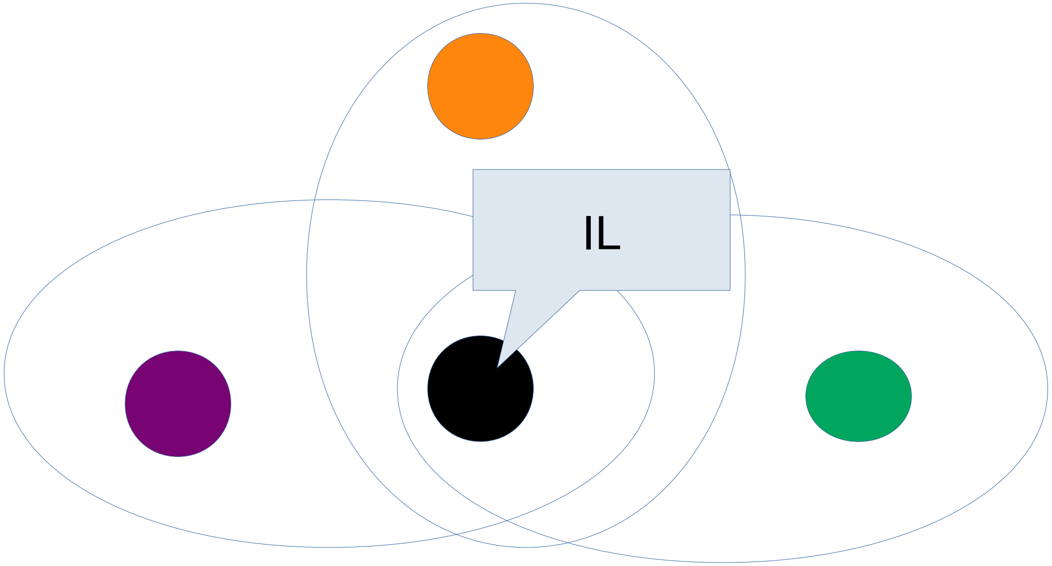
One minimalist foundation: FOL + ZF(C)

Compatible with (almost) all classical foundations and greatly expressive



The Constructive Zoo

Many incompatible foundations: IZF, CZF, Bishop, Topos theory, intuitionism, Russian, MLTT, Coq, HOTT, ...



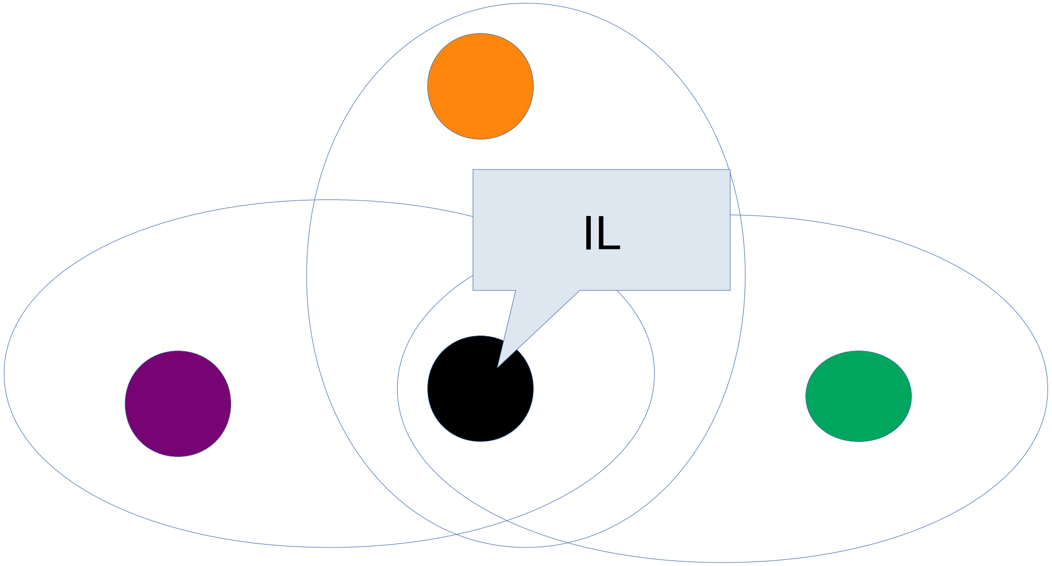
The Constructive Zoo

Example: the Cauchy reals can be

- computable only and you know it in the logic
- computable only, but you don't know it and you can assume they are not
- not computable
- strictly included in the Dedekind reals (which are not computable)
- isomorphic to the Dedekind reals
- forming a set vs forming a class (same for the Dedekind reals)

Towards MTT

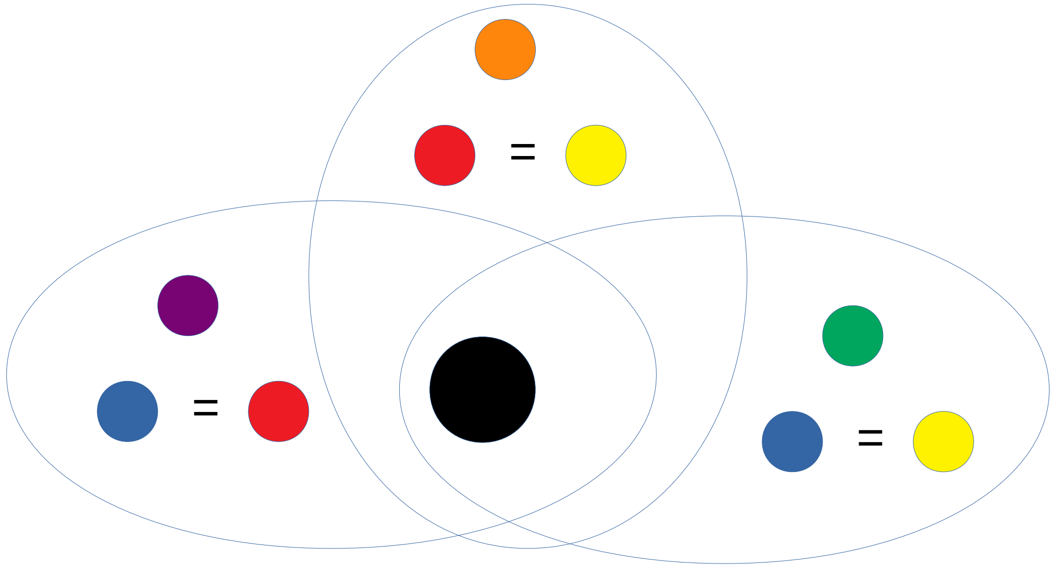
Intersection: inexpressive
Union: inconsistent



Towards MTT

MTT: preserve all differences

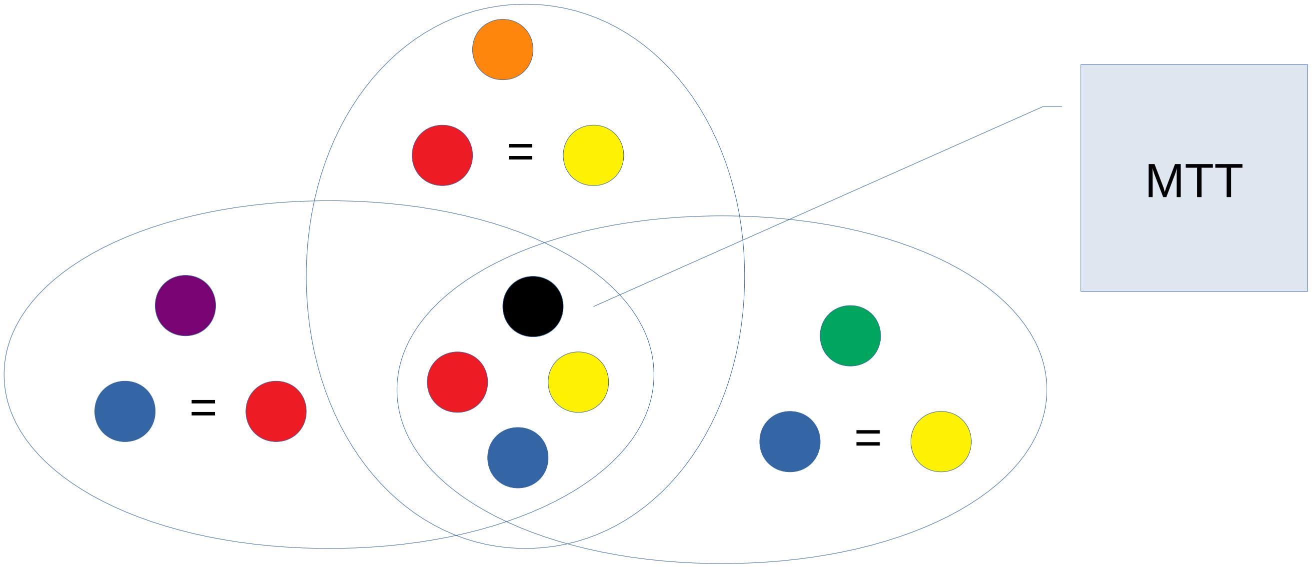
Other theories: collapse of concepts + new stuff



Towards MTT

MTT: preserve all differences

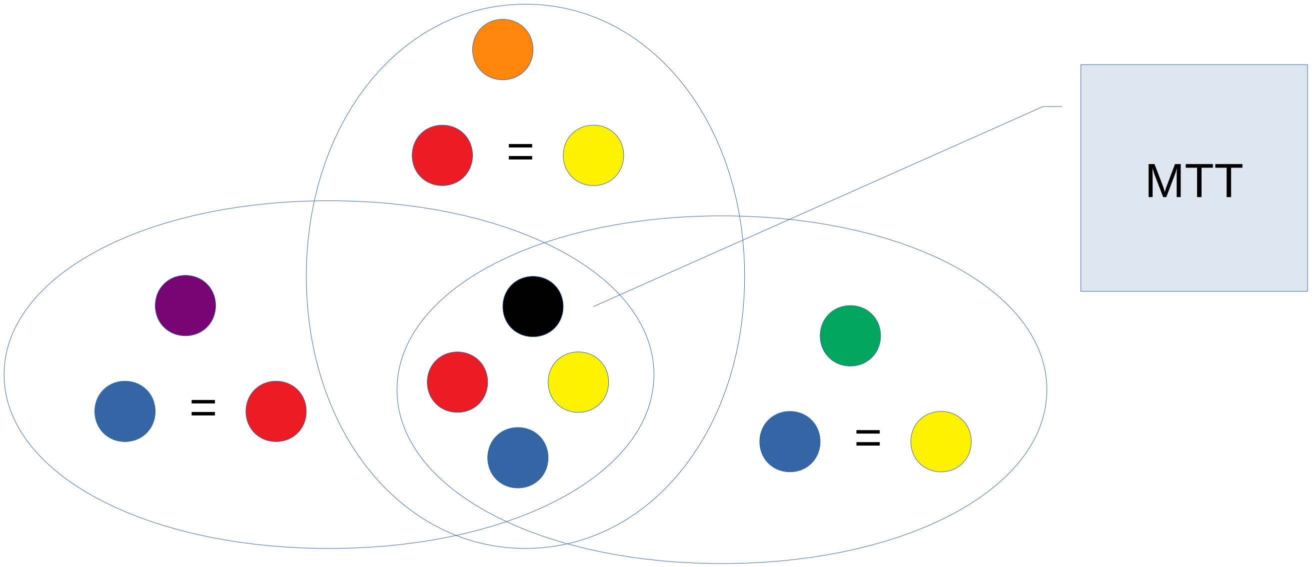
Other theories: collapse of concepts + new stuff



Towards MTT

MTT: compatible with all foundations

MTT: is it expressive enough?



Reals in MTT

- Terms of type A → B
  - computable (and you know it!), enumerable, form a set
  - set of computable, enumerable Cauchy reals

- Functions B^A i.e. terms (relations) of type A → B → Prop s.t. for each a:A there is exactly one b:B in relation
  - not known to be enumerable and computable (no axiom of unique choice!), form a class
  - class of Cauchy reals
  - class of Dedekind reals, contains the Cauchy reals up to isos

Reals in MTT

+ axiom of unique choice (= Bishop)
  - $A \to B == B^A$
+ axiom of EM (= classical math)
  - $A \to B$ computable, B^A not computable
+ power-set axiom
  - Cauchy/Dedekind reals form a set
+ ...

The Two Levels

The Two Layers

Extensional Level

- set-theory like
- no proof terms
- extensional (quotients)
- undecidable

- undecidable
- recover infor.
- impl. quotients

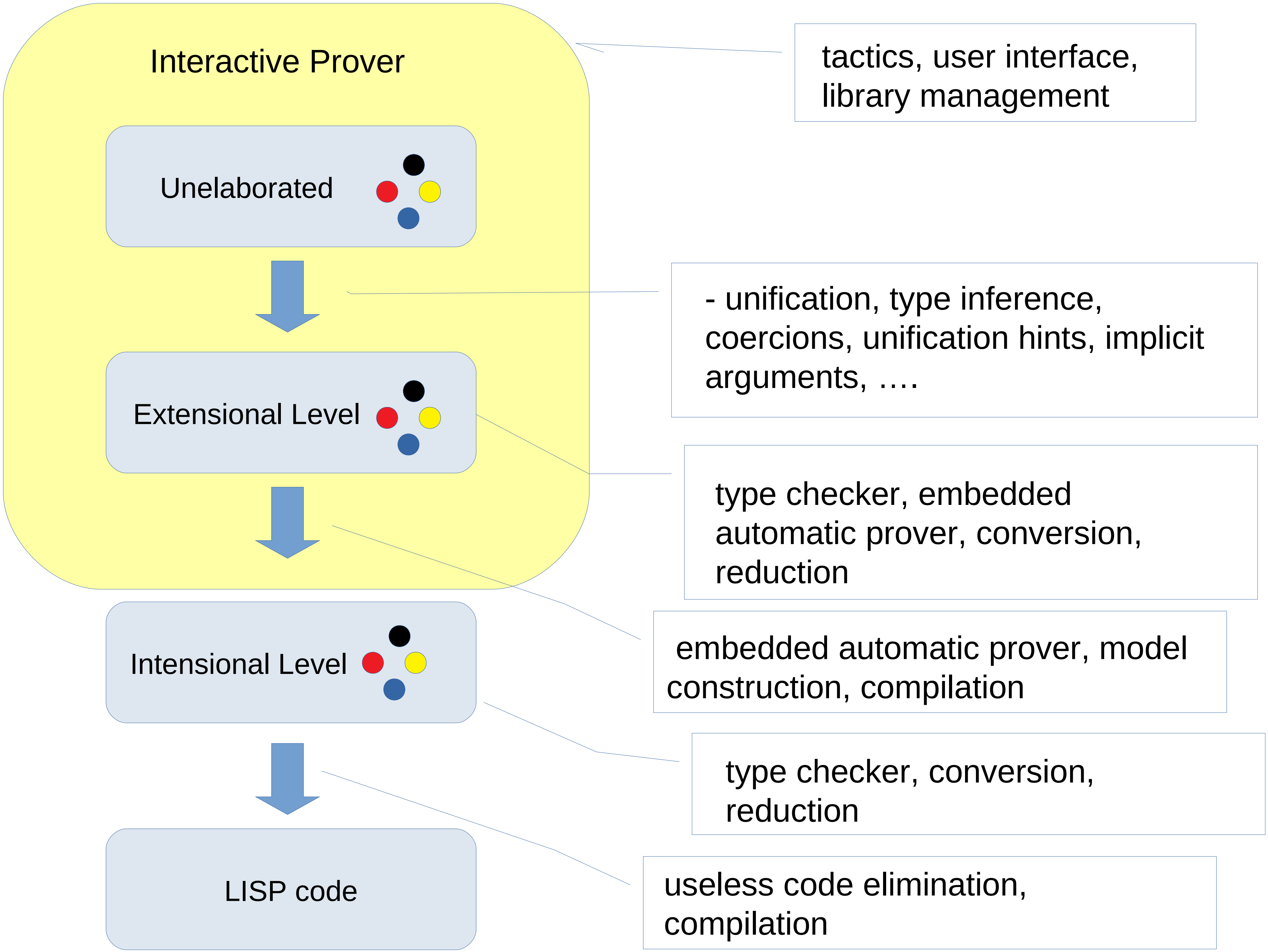
Intensional Level

- type-theory like
- proof terms
- intensional
- decidable

The Big Picture

a.k.a.

WIP (in LambdaProlog)



Interactive Prover
Unelaborated
Extensional Level
Intensional Level
LISP code
tactics, user interface, library management
- unification, type inference, coercions, unification hints, implicit arguments, ….
type checker, embedded automatic prover, conversion, reduction
embedded automatic prover, model construction, compilation
type checker, conversion, reduction
useless code elimination, compilation

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